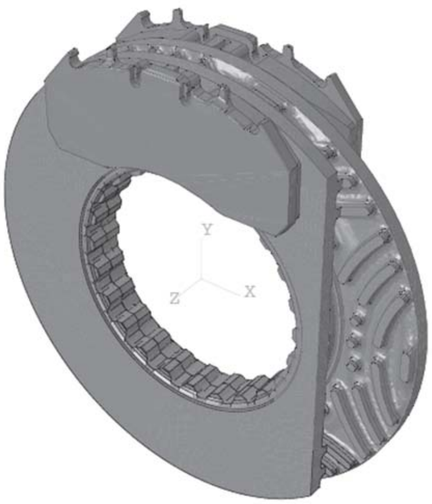
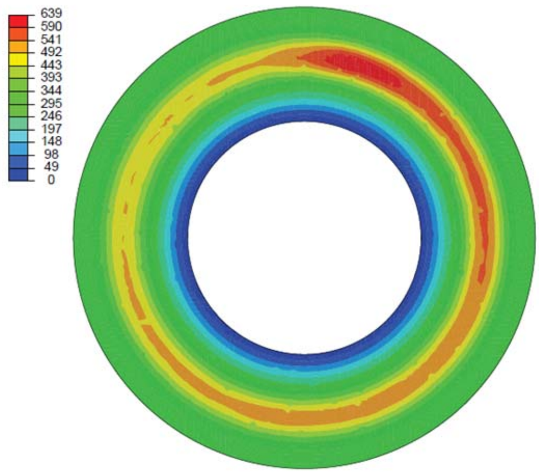
Thermo-mechanical Simulation and Optimisation of Disc Brake Systems





Project within < Fordons & Trafiksäkerhet>

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FFI

Content

FFI in short

FFI is a partnership between the Swedish government and automotive industry for joint funding of research, innovation and development concentrating on Climate & Environment and Safety. FFI has R&D activities worth approx. €100 million per year, of which half is governmental funding. The background to the investment is that development within road transportation and Swedish automotive industry has big impact for growth. FFI will contribute to the following main goals: Reducing the environmental impact of transport, reducing the number killed and injured in traffic and Strengthening international competitiveness. Currently there are five collaboration programs: **Vehicle Development, Transport Efficiency, Vehicle and Traffic Safety, Energy & Environment and Sustainable Production Technology.**

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FFI

1. Executive summary

The braking system is an important system in all vehicles and must function satisfactorily in all possible operating conditions. Disc brakes are common in these systems and is also currently becoming more common in brake systems for heavy vehicles. An important reason for this is that it is less complicated to adapt the automatic brake control for disc brakes compared to the traditional drum brake. When a vehicle decelerates a significant amount of power in the form of kinetic energy is transferred into heat via friction losses between the disc and the brake pad. This thermal energy leads to a temperature increase in the system, this increase can be several hundred degrees. High temperatures or many large temperature cycles may cause the system to lose braking ability or lose braking ability. The latter is obviously a process which may not occur in road traffic. The brake system must be sufficiently robust and reliable to rule out such scenarios. A well-defined process window for the brake system is necessary, especially in the use of automatic control. Today, the creation of such process window brake exclusively of natural experiments that are largely implemented in the brake rigging. These tests are expensive and take a long time to implement. It would be desirable if the experiments could be complemented by computer simulations. Unfortunately, the computation time is currently to extensive with the standard methods available in today's commercial software. The reason is in the established formulation (Lagrange) used for solid bodies, where the finite element mesh included with the body. In the case of the brake disc, it will be costly to rotate the mesh several turns in combination with solution of the thermo-mechanical friction problem.

One idea would be to formulate the disc instead with a fixed mesh (Eulerian) typically used in fluid mechanics, and, allowing the disc to rotate through the mesh. In this project, this idea has been used to develop a toolbox where heat dissipation in the brake disc and brake pad can be studied virtual, both efficiently and accurately. Said toolbox is developed for a three-dimensional geometry and includes contact, friction and wear between the disc and the brake pad. The frictional heat is dissipated by classical thermal conduction, and the disc and brake pad expands and deforms according to classical a thermo-elasticity approach. The temperature history from physical braking phase is simulated by using the toolbox and very good agreement is obtained. An interface to the commercial software ABAQUS is also developed. With this interface, the temperature history from the developed toolbox used in Abaqus to study the thermal residual stresses. For this purpose, a temperature dependent plasticity model has been developed that describes the mechanical properties of the brake disc at different temperatures. The simulations show that the positive tensile stresses in circumferentially are develop for repeated brake applications. Such stresses may explain the radial cracks that may develop in attempts at repeated hard braking.

This work has resulted in a Licentiate thesis, 6 publications and abstracts published in two conferences.

The professor responsible for this project initiated the project at Jönköping University (JTH). Because of a dispute between the management of JTH and the responsible professor the responsible professor ended his employment at JTH before the project was completed. Thus, not all project funds spent, and project objectives are not fully met. Despite the above-mentioned circumstances, the responsible professor succeeded in delivering project results in substantial quantities. The industrial party in this project are very content with said results. Part 2 of this project has been initiated at University West and follow the original plan (Rno nr. 2012-00046).

2. Background

Several "intelligent" active safety systems have been developed the last years and the development will most likely continue in the future. Many of these systems are using the foundation brake (the ordinary wheel brake) in various ways, controlled by algorithms using speed, slip, GPS, radar, laser etc. for data input. Examples of these functions are Electronic Brake System (EBS), Adaptive Cruise Control (ACC) where the driver set the desired speed on the highway and if a vehicle is too close in front of the vehicle the vehicle will brake automatically, queue support where the driver is assisted during inch in traffic jams, brake control algorithms etc.

Active safety systems using the foundation brake will subject the disc brake components to new load cycles (e.g. the thermal load pattern / thermal load cycle on the brake disc will be changed), this will result in new/modified design criteria. Therefore improved knowledge of the underlying mechanisms of thermal disc cracks, fatigue life etc. in the combination with active safety systems is needed in order to further develop brake components and to be able to fully utilise the full potential of future active safety systems. Several previous studies, Master thesis as well as project work at Volvo, has highlighted the importance of knowledge on the thermal aspects and brake performance and durability and how to what extent new active safety systems influence the design criterions of foundation brakes hardware.

Additionally the validation part within development process of a disc brake system is very cost intensive due to extensive rig tests. Due to the complexity of the disc brake system the rig tests can not reveal all details, which are very important for the improved understanding. Further on the effect of various active safety systems is difficult to test in ordinary test rigs.

The assembly of the disc-pad system considered in this work is shown in Figure 1, Left, with one disc sectioned to reveal the ventilation vanes (patented [I]). This is the assembly of a disc brake system of a heavy duty Volvo truck (however, all components of the brake system are not included in Figure 1).

The splines at the inner periphery of the disc are used to mount the disc to the wheel hub by engaging corresponding splines. For the simulation of frictional heating and stresses these splines are not considered important so they have been removed to simplify the model. Similarly some geometry of the back plate has been removed to simplify the model. The assembly with simplified geometries of the disc and the back plate is shown in Figure 1, Right.

Figure 1. Left: The assembly of the disc-pad system with a disc shown sectioned. Right: The assembly of the disc-pad system after removing the geometry not considered important for the simulation

3. Objective

The entire project relates to the heat generated during braking; prediction of the thermal increase, the consequences from increased temperature of components and increased cycling of temperatures and how can the thermal energy be handled from a design perspective. The internal temperature of the brake disc can be quickly increased, even for a low brake usage. The following can be considered: A braking that lasts 20 s duration at only 10 % of the maximum brake pressure, the braking procedure is repeated 5 times with 60 s cooling time between the braking applications; the initial temperature was 100 °C which is regarded as a typical working temperature. After the 1st brake application the brake disc temperature is 160 °C, after the 2nd 220 °C, 3rd 250 °C, 4th 340 °C after the 5th 400 °C. As stated above, these are internal disc temperatures, however, it is not the core of the brake disc which is prone to fatigue cracks, it is the surface of the disc. The temperature of said surface will be significantly higher for the braking procedure stated above, more precisely 625°C after the 5th iteration.

The last years of rapid improvements of computer power have opened up new possibilities for advanced computer based simulations. Today, the prevalent way to simulate frictional heating of disc brakes in commercial software is to use the fully coupled Lagrangian approach in which the finite element mesh of a disc rotates relative to a brake pad and, thermal and mechanical analysis are performed simultaneously.

Although this approach works well, it is not feasible due to extremely long computational times. To overcome the issue with long computational times a sequentially coupled approach has been adapted in this work where thermal and mechanical analyses are performed sequentially. An in-house software was developed which performs thermal analysis in an Eulerian framework, where the finite element mesh of the disc does not rotate relative to the brake pad but the material flows through the mesh. The temperature history from the frictional heat analysis is used as an input in a coupled stress analysis which is performed in the commercial software Abaqus. This sequential approach requires significantly lower computational time as compared to the Lagrangian approach.

The development of the software was divided into ***two parts***; In the ***first part***, the in-house software was developed to perform the thermal analysis of a brake disc. In this analysis, brake force is not applied suddenly instead it is ramped up by using a log-sigmoid function as shown in Figure 2 for a typical case.

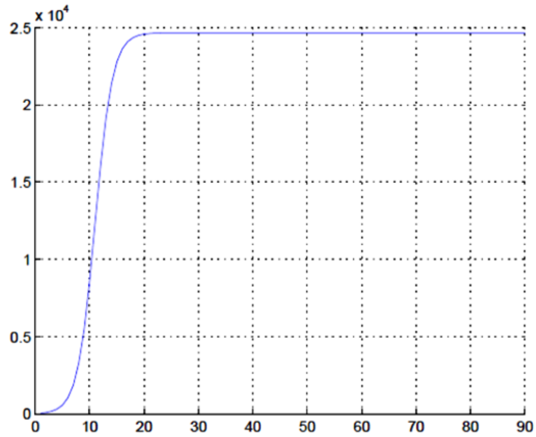

Figure 2. Ramping up of brake force during thermal analysis.

Due to the severe working conditions during operation, macro-cracks might develop on the disc surface in the radial direction [II, III]. Therefore it is important to study a disc for the existence of stresses and ultimately estimate its fatigue life. In the ***second part*** thermal stresses have been computed for the disc. The stress analysis results show that during hard braking, high compressive stresses are generated in the circumferential direction on the disc surface which causes plastic yielding. But when the disc cools down, these compressive stresses transform to tensile stresses. For repeated braking when this kind of stress-strain history is repeated, stress cycles with high amplitudes are developed which might generate low cycle fatigue cracks after a few braking cycles.

For the stress analysis, a frictional heat analysis was performed first. A ring of high temperatures, called hot band, can be distinguished in the middle of the disc surface. For this simulation, a brake force of 24.5 [kN] is applied for 20 [s] on the back surface of the support plate. The angular velocity of the disc is 45 [rad/s] and held constant throughout the simulation. The force is ramped up by using a log-sigmoid function during 20 time increments and then held constant for the next 80 increments with time step t=0.2 [s].

This loadcase corresponds to a truck moving downhill with a constant speed. The friction coefficient is μ = 0.3, the contact conductance is φ = 0.1 [W/NK] and the convection coefficient is set to 50 [W/m2K]. After braking, the disc is cooled for another 5000 [s] with a time step of 5 [s] in order to bring it back to the ambient temperature. The temperature history of this frictional heat analysis is imported to commercial software, Abaqus, and a stress analysis is performed. To perform stress analysis for repeated braking, it is assumed that braking conditions are same for all the brake cycles so they generate similar temperature history. From the results it can also be seen that the thermal strain has higher range than the mechanical strain, so it influences the total strain more. This can be explained by the temperature dependent thermal expansion coefficient. Such stress-strain cycles might generate radial cracks after a few braking operations resulting in low cycle fatigue of a disc brake.

After computing the stresses, if the fatigue life data for the disc material is known, its fatigue life can be assessed. It is clear that material on the disc surface undergoes substantial plastic strain, which suggests that a strain-based approach is needed for predicting the fatigue life. Such a study was performed while adapting the Coffin-Manson relation. The material parameter for this study was obtained from literature. The resulting thermal behavior and the form effect that the temperature increase has on brake components is detailed in the result section.

Wear of a pad might not play a big role while considering only one brake cycle but accumulated wear over several brake cycles certainly influences the temperature distribution and hence thermomechanical behavior of a brake disc. To study the wear procedure the pad a simulation case was built using the same parameter values (brake force, time of brake application, angular velocity, friction coefficient, wear coefficient, conductance coefficient and convective coefficient). In total, 40 cycles applications were made analyzing the wear/form alteration of the pad continuously. It was observed that the wear is concentrated in the middle of the pad thus enabling the formation of hot bands. By intuition it can be thought that wear should be higher near the outer radius of the disc. But it is concentrated approximately in the middle of the pad surface.

The temperatures predicted by the in-house software have been compared with the temperatures recorded by a thermal imaging camera during a physical test and found to be relatively higher as shown in Figure 3. Also, using the thermograph it was possible to validate the specific variation in temperature referred to as hot bands. As can be seen in the figure below two hot bands predicted in the simulations after repeated brake cycles were confirmed, however, the hot bands were not as distinct as observed in the thermograph.

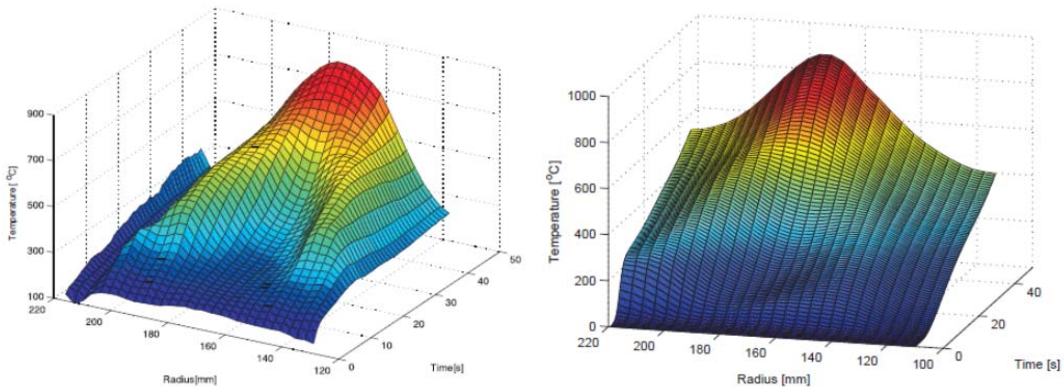

Figure 3. Temperature as a function of time and disc radius. Left: Result of thermal imaging camera measured in brake dynamometer. Right: Comparative simulation result

These differences could be due to temperature independent material data, friction coefficient, and wear coefficient used during the frictional heat analysis. For more realistic results, temperature dependent material data should be used, see the Conclusions and future work section for more details regarding this.

4. Project realization

4.1 General project description

Due to the project complexity the project was divided into four phases.

Phase one

In phase one the global analysis model of the disc brake system was created by modelling, defining boundary conditions etc. Correlation between simulated results and measured results will be done on a global level e.g. achieve the same thermal pattern on the brake disc surface, reaching comparable temperature levels in well-defined areas. In this part several parameters have been studied, such as the behaviour of the friction pair brake disc – brake pad, properties for the redistribution of the thermal load, conduction and convection. The complexity further increased due to the temperature dependency of the material characteristics. To be able to solve the FE calculations efficiently an in-house software was created, this software has then been further developed and used in all phases.

Phase two

Now when the global behaviour of the computer simulation was well correlated to what can be seen in the reality for physical tests Phase two will continued with building up detailed knowledge concerning frictional heating, thermal stresses, disc cracks and fatigue life, wear of disc and pads was also be estimated. For the complete picture also the most important manufacturing influences will be taken into account e.g. residual stresses. During a brake cycle such residual stresses from solidification might be relaxed resulting in un-symmetric deformation modes of the disc.

Phase three

With the deeper knowledge gained in Phase two, Phase three started with investigating the relationship between thermal stresses, material parameters, wear etc. in order to find out what should be optimised. Then the main focus will be on the optimisation of the most important parameters e.g. fatigue life. To achieve this response surface optimisation was be applied and further developed.

Phase four

Phase four was more or less only initiated in this project. The main work is intended for the continuation project, 2012-00046. However, in this project a few real life brake sequences were simulated in the brake simulation test rig. The brake simulation test rig (SIL) generated transient input data with the most significant parameters (vehicle speed, brake pressure, time stamp etc.) which was used in the thermal FE calculations.

5. Results and deliverables

The results and deliverables are divided into three sections, (1) Delivery to FFI-goals which details the results according to the aims of the < Fordons & Trafiksäkerhet > program, (2) Delivery according to the general project description which details the results according to the aims of the project description and (3) Increasing the accuracy and the effectiveness of foundation brake simulations which provides a snapshot of the results that demonstrates the highest academic relevance.

5.1 Delivery to FFI-goals

This project will contribute to some of the targets set out for the < Fordons & Trafiksäkerhet > program. In the long run the gained knowledge in this project can result in improved brake performance (e.g. shorter stopping distance)

The Volvo experience regarding active safety systems using the foundation brake is that automatic systems will generate another type of energy/temperature distribution in the brake system compared to “manual” braking by experienced drivers. Accordingly; the mechanical components that are used today are designed to operate for a load spectrum of today. Active safety systems might influence the load spectrum in a negative way for the mechanical components; In order to illustrate this let’s assume that an ACC system will result in an increased future load spectrum. To enable coverage of said increased loads one would choose between two paths, redesign disc brake components and surrounding components in order to improve their cooling performance with the corresponding amount as the temperature increase caused by the higher load or learn more about the different phenomena that are affected by energy/temperature and influencing the brake performance and brake characteristics. The first path offers more limited improvements but it could of course be combined with the second path. If the second path is followed one can really learn and understand the behaviour of the different phenomena such as disc cracks, thermal fatigue etc. With that understanding the design can be adopted to better cover these aspects and create more energy/temperature resistant designs. Consequently, the main result of this project has been increasing the knowledge of how to design more robust and reliable disc brake components in order to increase the level for average braking characteristics and to create the necessary margins for brake energy and temperatures in order to allow more foundation brake usage in the different active safety systems that will be developed in the future.

The knowledge gained within this project will serve as the foundation for coming active safety systems where the foundation brakes are utilised. Consequently the knowledge from this project will open up the possibility to new or improved active safety systems e.g. EBS, ESP, ACC, queue support, etc.

This project has served as a very important piece in building up the co-operation between the industry and the university concerning computer based disc brake system analysis and virtual validation of disc brake systems coupled to active safety systems.

Virtual testing of disc brake cycles by using nonlinear FEA has been performed. It is clear that thermomechanical analysis by FEA can be used today to simulate different phenomena appearing in a brake cycle, e.g. see the discussion above on recent results on simulation of TEI.

A long run goal in this and future projects is to initiate some work on optimisation driven design of disc brake systems. The traditional design process can be termed iterative-intuitive: while new designs are typically analysed at high precision by advanced FEA, the re-design is instead made manually (intuitively) without computational indications of trends and consequences. In contrast, when using optimisation driven design, the re-designs are found as solutions or outcomes from precisely formulated optimisation problems.

5.2 Delivery according to the general project description

Since the scope of this project neighbours a possible future project both project are included in this summary.

”Base” project (Rno. 2009-00086))

- Basic research, increase the competence level → **Base platform**
- Numerical platform, simulation toolbox for robust and efficient simulations

Continuation project (Rno. 2012-00046))

- Detailed parameter and mechanism studies
- Improve the coupling to brake system simulation rigs
- **Implement** the developed methods and knowledge in the design & validation process

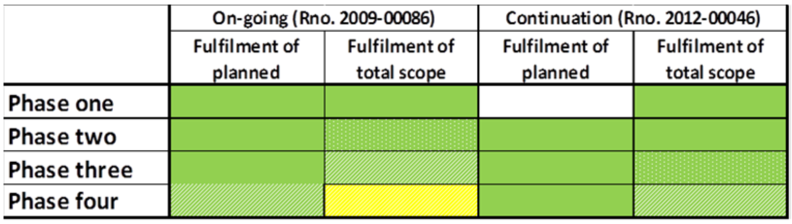

Figure 4. Overview of progress according to the four phases described in the project realization

5.3 Increasing the accuracy and the effectiveness of foundation brake simulations

A complete simulation, i.e. thermal analysis and stress analysis, requires a fraction of time as compared to the same analysis performed in commercial software based on Lagrangian framework has been developed in this project. The stress analysis results show that during hard braking, high compressive stresses are generated in the circumferential direction on the disc surface which causes plastic yielding. But when the disc cools down, these compressive stresses transform to tensile stresses. For repeated braking when this kind of stress-strain history is repeated, stress cycles with high amplitudes are developed which might generate low cycle fatigue cracks after a few braking cycles.

Figure 5 exemplifies the stresses in the brake disc, in said figure a graph of circumferential stresses against different measures of the strain in circumferential direction for the repeated braking is shown. It can be observed that the thermal strain has higher range than the mechanical strain ($\varepsilon^e+\varepsilon^p$), so it influences the graph of the total strain more. The shape of the graphs can be explained by the temperature dependent thermal expansion coefficient and the von Mises material model. Such stress-strain cycles might generate radial cracks after a few braking operations resulting in low cycle fatigue of a disc brake.

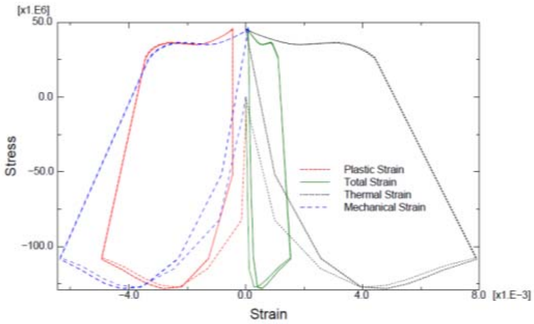

Figure 5. Four different stress-strain graphs for three repeated braking operations.

It is important to realize the effectiveness of the developed simulation procedure, for this report the calculation time needed to complete the simulation in section is used as an example of said effectiveness. The total CPU time for the above mentioned frictional heat and stress analysis is 3.05 [h] and 5.54 [h] respectively, on a workstation with Intel Xeon X5672 3.20 GHz processor. For comparison purpose, a simulation was performed for one brake cycle in commercial software on the same workstation using a total Lagrangian framework. The brake force was applied only for 0.1 [s]. Otherwise its magnitude and the boundary conditions were the same as used in our sequential approach. After braking the disc was not cooled down. The total CPU time for the analysis is 40.12 [h]. This shows that a tremendous amount of computational time is required for the total Lagrangian approach even though the analysis was run for a braking period of only 0.1 [s]. In order to simulate the braking of 20 [s] studied in this work, approximately 8024 [h] will be needed if a total Lagrangian approach is applied.

6. Dissemination and publications

6.1 Knowledge and results dissemination

Seen from both an academic and industrial point of view, the result from this calculation developed brake very useful. To date it has been possible to verify the result of calculations in industrial braking test which is performed in the brake dynamometer in Gothenburg. It has namely been able to reproduce the phenomenon in the brake calculation features include quantified using infrared camera in the actual test (see Figure 3).

Using the developed code, and combining this with the design of experiments (which practically is only realizable when the computation time has been significantly reduced) and using specific optimization techniques will it will in the near future be possible to design brakes that provide maximum performance (fatigue resistance and brake power) and a minimized weight. The developed calculation method reproduces a detailed picture of the temperature variations, to be able to reproduce the temperature variations in an illustrative way, a direct knowledge-building form which is desirable in product

development, because several reasons, the temperature variation in a physical test may be difficult to reproduce with the same complexity / detail.
Volvo has had direct knowledge exchange with the performing party in the form of:

- Training sessions where the performing party has practically and theoretically shown how the developed software is used. The developed software has been used in Volvo, the current situation is in a testing phase. Exchange in the form of test results from advanced component testing (brake dynamometer) and discussion of the underlying causes that control temperatures in brake components has been a part of the project. This has increased the theoretical knowledge in the Volvo to a much more detailed level.

- In addition to the exchange of knowledge within Volvo, Niclas Strömberg represented the project at several conferences and seminars, and supervised a number of graduate students

6.2 Publications

M. Hofwing, Robustness of residual stresses in brake discs by meta modeling, in the proceedings of IDETC/CIE 2011, ASME, Washington, 2011.

N. Strömberg, Development and Implementation of an Eulerian Approach for Efficient Simulation of Frictional Heating in Sliding Contacts, in the proceedings of the IV International Conference on Computational Methods for Coupled Problems in Science and Engineering, Eccomas, 20-22 June, Kos, Greece, 2011.

N. Strömberg, An Eulerian Approach for Simulating Frictional Heating in Disc-Pad Systems, European Journal of Mechanics, A/Solids, 30, 673683, 2011.

N. Strömberg, Simulering av bromsvärme i en skivbroms med en Eulerformulering, Svenska Mekanikdagar, June 13-15, Gothenburg, Sweden, 2011.

N. Strömberg, An Eulerian approach for simulating frictional heat bands in rotating discs, Euromech 514: New trends in contact mechanics, 27-31 March, Corsica, France, 2012.

N. Strömberg & A. Rashid, An Efficient Sequential Approach for Simulation of Thermal Stresses in Disc Brakes, in the proceedings of the 15th Nordic Symposium on Tribology, NORDTRIB, Trondheim, Norway, 1215 June, 2012.

A. Rashid & N. Strömberg, Sequential Simulation of Thermal Stresses in Disc Brakes for Repeated Braking, Journal of Engineering Tribology, accepted, 2013.

A. Rashid & N. Strömberg, Thermomechanical Simulation of Wear and Hot Bands in a Disc Brake by adopting an Eulerian approach, in the proceedings of Eurobrake 2013, 1719 June, Dresden, Germany, 2013.

A. Rashid, Simulation of Thermal Stresses in a Brake Disc, Licentiate Thesis No. 1603, LIU-TEK-LIC-2013:37, Linköping, 2013.

7. Conclusions and future research

In the second phase thermal stresses have been computed for the disc. A complete simulation, i.e. thermal analysis and stress analysis, requires a fraction of time as compared to the same analysis performed in commercial software based on Lagrangian framework. The stress analysis results show that during hard braking, high compressive stresses are generated in the circumferential direction on the disc surface which causes plastic yielding. But when the disc cools down, these compressive stresses transform to tensile stresses. For repeated braking when this kind of stress-strain history is repeated, stress cycles with high amplitudes are developed which might generate low cycle fatigue cracks after a few braking cycles.

The main item of this project has been increasing the understanding of the thermal properties of the brake disc/brake pad interface and the brake disc, it would be desirable to include more components in the such as the the splines at the inner periphery of the disc and also a more detailed model of the back plate. In a future simulation model it would be desirable to include neighbouring component such as the hub and the wheel end bearing, both to obtain a more detailed model of the feat flux and also the possibility to study thermal characteristics on neighbouring components that the foundation brake influences.

In this study the braking layout was simplified (as e.g. mentioned in the background section) using a constant frequency of 20 s braking and a 60 s ambient duration (cooling). This is not an accurate depiction of a braking cycle, a given next step would be to use a more complicated cycle recorded from an existing field vehicle. Said analysis might require an adaption of the iteration algorithms, more specifically the time increment/time step for a brake application and cooling cycle.

Regarding the parameters used in the simulation tool, the following items needs more attention:

The friction coefficient of a brake pad is generally dependent on temperature, velocity and contact pressure [IV] but in this simulation it is assumed to be constant at $\mu = 0.3$ to represent an average behavior. Similarly, the wear coefficient is generally dependent on temperature and velocity [V, VI] but in this work it is assumed to be constant at 10^{-10} [m^2/N]. The wear coefficient is only estimated, however, wear coefficient is strongly dependent on temperature especially at high temperatures [V, VI]. In future a representative value of wear coefficient for a given pad should be determined and its dependence on temperature should also be taken into account. Furthermore convective coefficient is assumed to be same for all surfaces which are quite an unrealistic assumption.

In the stress analysis, a material model has been used that assumes the same behaviour of the material both in tension and compression but in reality cast iron has different properties in tension and compression. So in the future it would be advantageous to implement a material model incorporating the different behaviour of cast iron in tension and compression. Furthermost cast iron should also investigate for its fatigue data. This data is necessary to estimate fatigue life of the disc realistically.

8. Participating parties and contact persons

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